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Optimal reconciliation with immutable forecasts from specific hierarchies

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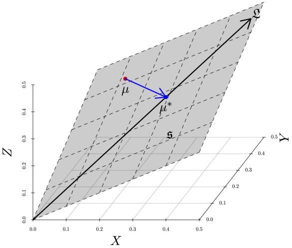
Forecast reconciliation



Panagiotelis et al. (2021)

- produces coherent forecasts,
- supports aligned decision making, and
- improves forecasting accuracy.

Motivation

Conventional forecast reconciliation methods

- may affect forecasting accuracy of certain levels (e.g., the top level) that previously perform "better", and
- are not suitable for specific scenarios such as fixed budget or purchase restrictions.

Our method

- keeps base forecasts of specific levels or nodes immutable after reconciliation, and
- is similar to top-down, middle-out and bottom-up methods, but more general and superior.

Outline

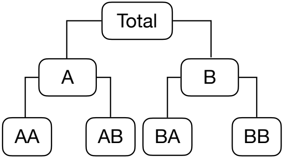
Notation

- $\hat{\boldsymbol{y}}_T(h)$: h-step-ahead base forecasts given observations up to T.
- $\tilde{\boldsymbol{y}}_T(h)$: h-step-ahead reconciled forecasts.
- $\boldsymbol{b}_t$: bottom-level time series at time t.
- $\boldsymbol{S}$: summing matrix, $\boldsymbol{y}_T(h) = \boldsymbol{S}\boldsymbol{b}_t$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{I}_m \end{bmatrix}_{n \times m},$$

where m is the number of bottom-level time series. n is the total number of time series in the hierarchy.

Notation Example



$$\boldsymbol{b}_t = \begin{bmatrix} y_{AA,t} \\ y_{AB,t} \\ y_{BA,t} \\ y_{BB,t} \end{bmatrix}$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{I}_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ \multicolumn{4}{c}{\boldsymbol{I}_4} \end{bmatrix}$$

$$\boldsymbol{y}_t = \begin{bmatrix} y_{Total,t} \\ y_{A,t} \\ y_{B,t} \\ y_{AA,t} \\ y_{AB,t} \\ y_{BA,t} \\ y_{BB,t} \end{bmatrix}$$

Forecast reconciliation through trace minimization (MinT)

- Let
$$\tilde{\boldsymbol{e}}_T(h) = \boldsymbol{y}_T(h) - \tilde{\boldsymbol{y}}_T(h),$$
be the reconciled forecast error.

- The objective of MinT method (Wickramasuriya et al. 2019) is
$$\begin{aligned} &\min \quad \operatorname{tr}\left[\operatorname{Var}[\tilde{\boldsymbol{e}}_T(h) \mid \mathcal{I}_T]\right] \\ = &\min_{\boldsymbol{P}} \quad \operatorname{tr}\left[\boldsymbol{S}\boldsymbol{P}\boldsymbol{W}_h\boldsymbol{P}'\boldsymbol{S}'\right], \end{aligned}$$
where $\boldsymbol{W}_h$ is the h-step-ahead base forecasts error.

Forecast reconciliation with equality constraints

- The new MinT problem is subjective to the following constraints:
  1. coherency,
  2. immutability.
- We formulate it into a linear equality constrained least squares problem:

$$\min_{\tilde{\boldsymbol{y}}_T(h)} \frac{1}{2}[\hat{\boldsymbol{y}}_T(h) - \tilde{\boldsymbol{y}}_T(h)]' \boldsymbol{W}_h^{-1} [\hat{\boldsymbol{y}}_T(h) - \tilde{\boldsymbol{y}}_T(h)]$$

$$\text{s. t.} \, \breve{\boldsymbol{\mathcal{U}}}' \tilde{\boldsymbol{y}}_T(h) - \boldsymbol{d} = \boldsymbol{0}.$$

Forecast reconciliation with equality constraints

$$\min_{\tilde{\boldsymbol{y}}_T(h)} \frac{1}{2}[\hat{\boldsymbol{y}}_T(h) - \tilde{\boldsymbol{y}}_T(h)]' \boldsymbol{W}_h^{-1} [\hat{\boldsymbol{y}}_T(h) - \tilde{\boldsymbol{y}}_T(h)]$$

$$\text{s. t.} \check{\mathcal{U}}' \tilde{\boldsymbol{y}}_T(h) - \boldsymbol{d} = \boldsymbol{0},$$

where

- $\check{\mathcal{U}}' = \begin{bmatrix} \boldsymbol{I}_n^{[\mathcal{A}]} \\ \boldsymbol{I}_{n-m} \quad -\boldsymbol{C} \end{bmatrix}$, $\boldsymbol{d} = \begin{bmatrix} \hat{\boldsymbol{y}}_T(h)^{[\mathcal{A}]} \\ \boldsymbol{0} \end{bmatrix}$.
- $\mathcal{A}$ is set of the immutable nodes, e.g. $\mathcal{A} = \{1\}$. $[\cdot]$ represents row subset.
- $\boldsymbol{W}_h$ is the covariance matrix of h-step-ahead reconciled forecast error.

Example

- Set the total level immutable during reconciliation,

$$\mathcal{A} = \{1\}.$$

- The constraint equations are

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} \hat{y}_{Total} \\ \hat{y}_{AA} \\ \hat{y}_{BB} \\ \hat{\boldsymbol{b}} \end{bmatrix} - \begin{bmatrix} \hat{y}_{Total} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\check{\mathcal{U}}' \hat{\boldsymbol{y}} - \boldsymbol{d} = \boldsymbol{0},$$

which ensures both coherency constraints and immutability constraint.

GLS estimator

We can derive the GLS estimator:

$$\tilde{\boldsymbol{y}}_T(h) = \hat{\boldsymbol{y}}_T(h) - \boldsymbol{W}_h\check{\mathcal{U}}(\check{\mathcal{U}}'\boldsymbol{W}_h\check{\mathcal{U}})^{-1}[\check{\mathcal{U}}'\hat{\boldsymbol{y}}_T(h) - \boldsymbol{d}].$$

Advantages of our method

- Combining classical top-down or middle-out method with forecast reconciliation.
- Producing **coherent** and **unbiased** and potentially more **accurate** forecasts.
- Advoiding unexpected accuracy decrease of specific levels.

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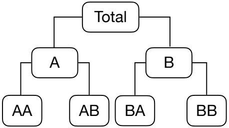
Monte Carlo simulations

We consider two scenarios:

1. The top-level use ETS base forecasts while other levels use ARIMA base forecasts.
2. The disaggregated levels are noisier, while aggregated levels are smooth due to aggregation.

Simulation setup



- Simulating bottom-level series using basic structural time series model, i.e., $b_t = \mu_t + s_t + \varepsilon_t$.
- Considering four estimators of W_h: OLS, WLS_s, WLS_v and MinT(Shrinkage), see Wickramasuriya et al. (2019).
- Comparing conventional FR and FR with immutable top level.
- Comparing results using ETS for all levels with results using ETS for top level and ARIMA for other levels.
- Repeating 1000 times and average RMSE reported.

Scenario 1

ETS

Level	Base	OLS U	OLS C	WLS_s U	WLS_s C	WLS_v U	WLS_v C	MinT(Shrinkage) U	MinT(Shrinkage) C
0	21.4003	21.3741	21.4003	21.3893	21.4003	21.4204	21.4003	21.4006	21.4003
1	11.9528	11.9269	11.9393	11.9255	11.9316	11.9347	11.9270	11.9165	11.9176
2	7.1655	7.1424	7.1474	7.1418	7.1440	7.1454	7.1417	7.1398	7.1392
Average	13.5062	13.4811	13.4956	13.4855	13.4920	13.5002	13.4897	13.4856	13.4857

ETS + ARIMA

Level	Base	OLS U	OLS C	WLS_s U	WLS_s C	WLS_v U	WLS_v C	MinT(Shrinkage) U	MinT(Shrinkage) C
0	21.4003	21.4422	21.4003	21.6658	21.4003	21.9257	21.4003	22.1046	21.4003
1	12.6553	12.1994	12.1921	12.2582	12.1625	12.3544	12.1495	12.4509	12.1726
2	7.7049	7.5549	7.5562	7.5745	7.5440	7.6079	7.5359	7.6255	7.5303
Average	13.9202	13.7322	13.7162	13.8328	13.7022	13.9627	13.6952	14.0603	13.7010

U: unconstrained(conventional FR). C: constrained(FR with immutable top level).

- The conventional FR methods may decrease the forecasting accuracy of top level.
- Keeping top level immutable shows extra improvements.

Scenario 2

ETS

Level	Base	OLS U	OLS C	WLS_s U	WLS_s C	WLS_v U	WLS_v C	MinT(Shrinkage) U	MinT(Shrinkage) C
0	21.2092	21.2327	21.2092	21.3476	21.2092	21.4054	21.2092	21.2712	21.2092
1	12.6115	12.5261	12.5133	12.5492	12.4857	12.5714	12.4828	12.4587	12.4286
2	8.4868	8.3818	8.3766	8.3910	8.3665	8.3999	8.3664	8.3571	8.3461
Average	14.1025	14.0469	14.0330	14.0959	14.0205	14.1256	14.0195	14.0290	13.9946

ETS + ARIMA

Level	Base	OLS U	OLS C	WLS_s U	WLS_s C	WLS_v U	WLS_v C	MinT(Shrinkage) U	MinT(Shrinkage) C
0	21.2092	21.3563	21.2092	21.6417	21.2092	21.8059	21.2092	21.9663	21.2092
1	13.2206	12.7756	12.7188	12.8465	12.6751	12.9120	12.6724	12.9523	12.6451
2	8.8454	8.6479	8.6287	8.6732	8.6126	8.6979	8.6131	8.7076	8.5982
Average	14.4251	14.2599	14.1856	14.3871	14.1656	14.4720	14.1649	14.5421	14.1509

U: unconstrained(conventional FR). C: constrained(FR with immutable top level).

Constrained forecast reconciliation shows more significant improvements in accuracy compared to unconstrained forecast reconciliation.

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Application to Wikipedia pageviews

The dataset contains 792 pageviews time series from 2016-06-01 to 2017-06-29 of the most popular social network ariticles on Wikipedia.

Table: Social Networking Wikipedia pageviews grouping structure.

Grouping	Series	Grouping	Series
Total		Language	
	1. Social Network		10. zh(Chinese)
Access		Purpose	
	2. Desktop		11. Blogging related
	3. Mobile app		12. Business
	4. Mobile web		13. Gaming
Agent			14. General purpose
	5. Spider		15. Life style
	6. User		16. Photo sharing
Language			17. Reunion
	7. en(English)		18. Video
	8. de(German)		
	9. es(Spanish)		

Ashouri et al.(2021)

Forecasting Results

ETS

Level	Base	OLS U	OLS C	WLS_s U	WLS_s C	WLS_v U	WLS_v C	MinT(Shrinkage) U	MinT(Shrinkage) C
Total	13626.016	14119.064	13626.016	14927.495	13626.016	15203.050	13626.016	18391.420	13626.016
Language	3522.352	3985.558	4002.137	4136.041	3991.678	4296.086	3835.849	5087.308	3943.314
Access	5992.494	6298.146	6412.368	6355.175	6080.896	6795.726	6188.317	8317.874	6786.659
Agent	9680.261	9631.388	9690.170	9085.663	8870.602	9032.231	8549.856	10629.915	9288.413
Purpose	3168.860	3160.098	3354.275	2756.765	2986.613	2713.131	2632.592	2809.312	2523.606
Network	604.998	715.276	736.057	585.550	611.558	530.573	519.708	599.078	568.955
Bottom	77.069	131.349	134.181	93.974	96.064	84.409	82.735	92.502	90.242
Average	5238.864	5434.411	5422.172	5420.095	5180.490	5522.172	5062.153	6561.059	5261.029

ETS + ARIMA

Level	Base	OLS U	OLS C	WLS_s U	WLS_s C	WLS_v U	WLS_v C	MinT(Shrinkage) U	MinT(Shrinkage) C
Total	13626.016	19632.850	13626.016	24382.092	13626.016	24759.578	13626.016	22525.647	13626.016
Language	5894.102	5656.007	6129.643	6347.531	5050.087	6452.627	4037.152	5903.984	3919.760
Access	8411.005	8050.464	7696.439	8986.060	6267.475	9147.077	5230.675	8055.827	5350.250
Agent	12826.847	11668.495	11587.060	12626.505	10274.044	12688.413	8247.357	11594.618	8116.384
Purpose	4090.334	4244.803	4872.881	3383.904	3229.584	3256.780	2428.806	3063.233	2398.055
Network	562.759	640.701	723.236	513.756	576.653	480.117	425.726	456.602	404.196
Bottom	72.006	113.973	127.878	84.849	92.143	71.974	72.143	68.007	68.396
Average	6497.581	7143.899	6394.736	8046.385	5588.000	8122.367	4866.839	7381.131	4840.437

U: unconstrained(conventional FR). C: constrained(FR with immutable top level).

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Demo of pyhts

Install pyhts:

```
pip install pyhts
```

Define the hierarchy structure:

```
from pyhts.hierarchy import Hierarchy
ht = Hierarchy.from_node_list(nodes=[[2], [2, 2]],
                              period=7)
```

Define a MinT(Shrinkage) model:

```
from pyhts.HFModel import HFModel
model = HFModel(hierarchy=ht, base_forecasters="arima",
                hf_method="comb", comb_method="mint",
                weights="shrinkage")
```

Demo of pyhts

Define a constrained MinT(Shrinkage) model:

```
from pyhts.HFModel import HFModel
model = HFModel(hierarchy=ht, base_forecasters="arima",
                hf_method="comb", comb_method="mint",
                weights="shrinkage", constraint_level=0)
```

Fit the model and generate reconciled forecasts:

```
import numpy as np
train = np.random.rand(400).reshape(100, 4)
test = np.random.rand(28).reshape(7, 4)
model.fit(train)
forecasts = model.predict(horizon=7)
```

Demo of pyhts

Evalute the forecasting accuracy:

```
# accuracy of reconciled forecasts
ht.accuracy(test, forecasts, hist=train,
            measure=['mase', 'rmse'])

# accuracy of base forecasts
base_forecasts = model.generate_base_forecast(horizon=12)
ht.accuracy_base(test, base_forecasts,
                 hist=train, measure=['mase', 'rmse'])
```

Outline

Conclusions

1. We propose a forecast reconciliation approach with equality constraints which can keep forecasts from specific levels and nodes immutable during reconciliation.
2. The proposed method is unbiased, flexible and potentially more accurate.
3. Monte Carlo simulations and application to Wikipedia pageviews dataset show its superiority over the state of art forecast reconciliation methods.

Thanks!

Paper: Coming soon on arXiv!
Package: `https://github.com/AngelPone/pyhts`
Documentation: `https://angelpone.github.io/pyhts/`